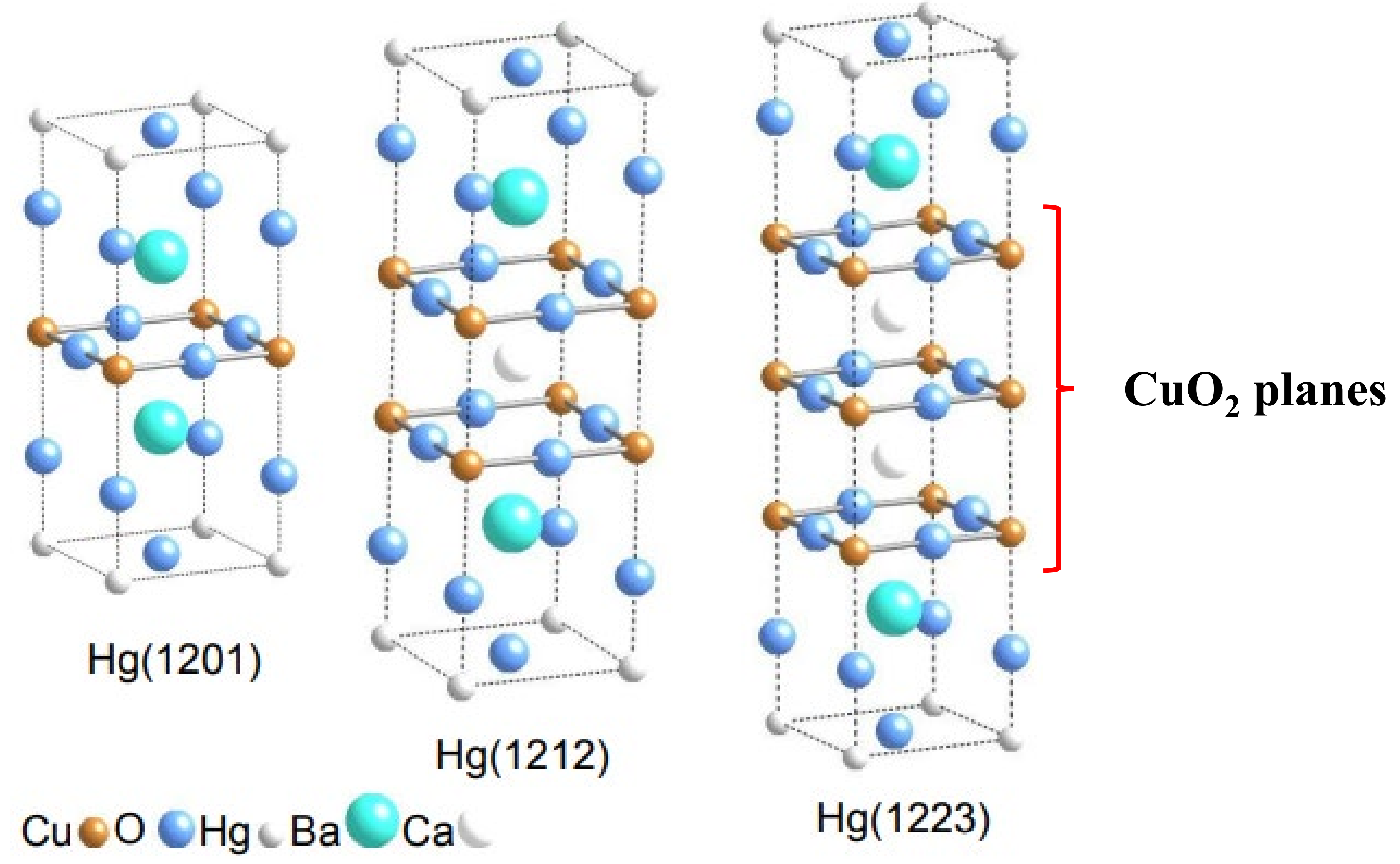
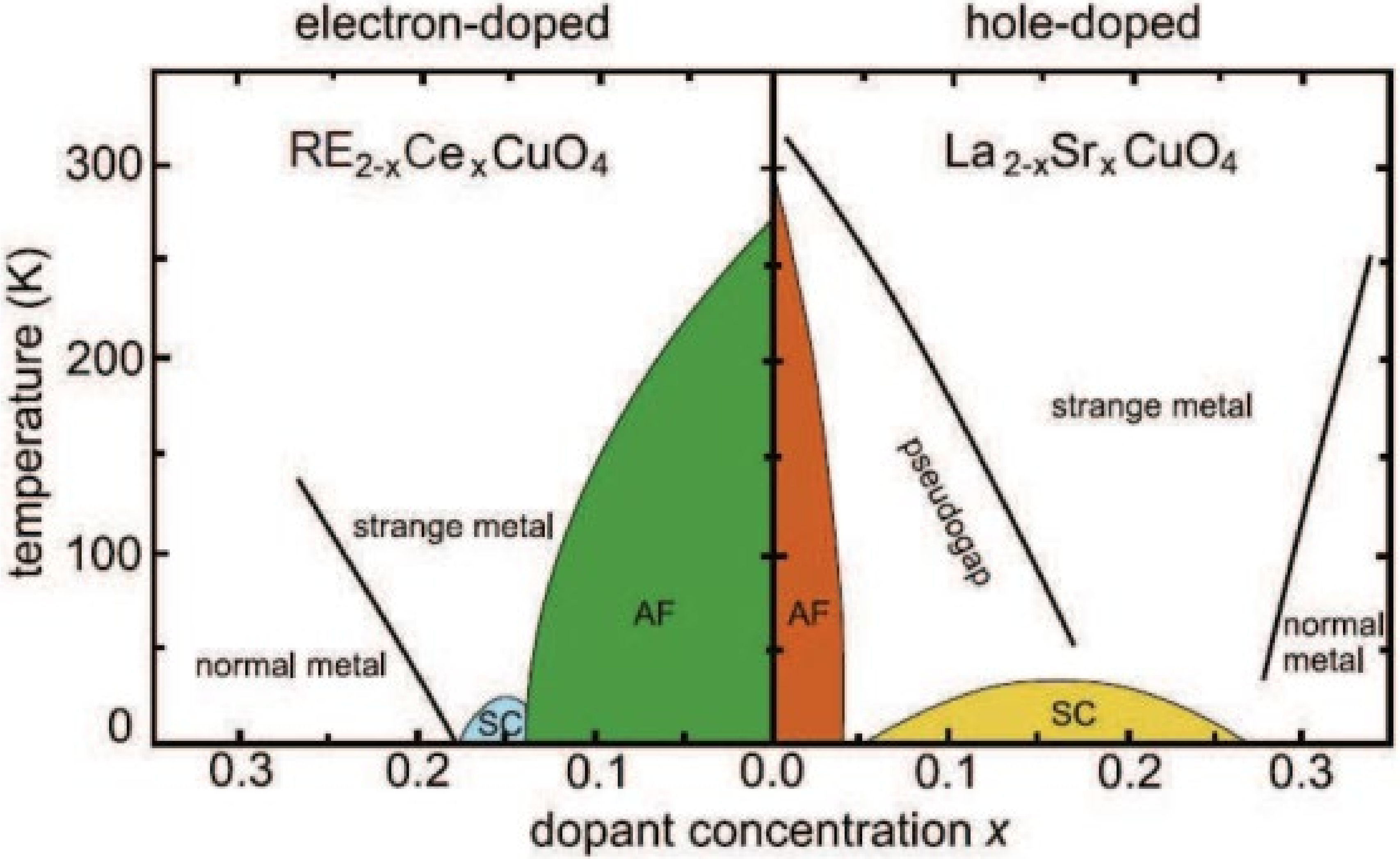
# Cuprate Superconductor Crystal Structures




D. J. Scalapino, "A common thread: The pairing interaction for unconventional superconductors," Rev Mod Phys **84 (4), 1383-1417 (2012).**

# Cuprate Superconductor Phase Diagram



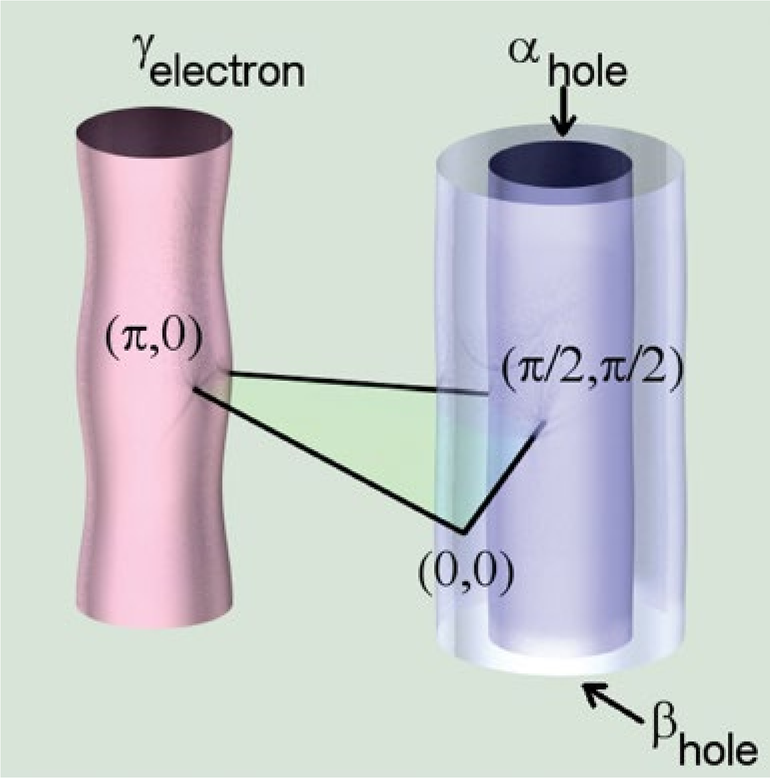

D. J. Scalapino, "A common thread: The pairing interaction for unconventional superconductors," Rev Mod Phys **84 (4), 1383-1417 (2012).**

# Cuprate Superconductor Basic Properties

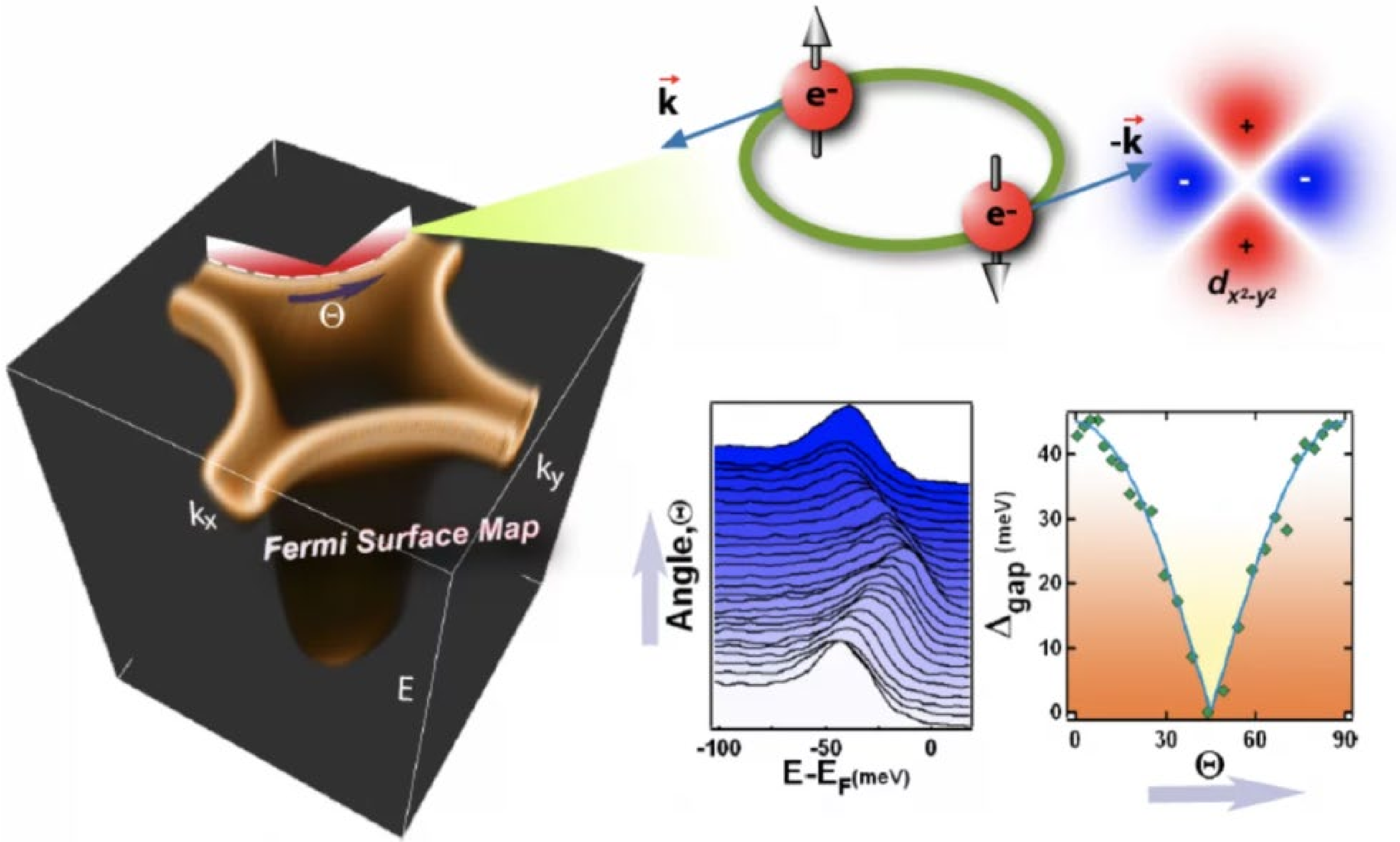
It was demonstrated through flux quantization measurements by Colin Gough that the cuprates have paired charge carriers, giving rise to flux quantization in units of $\Phi_0 = \frac{h}{2e}$. [C. E. Gough, M. S. Colclough, E. M. Forgan, R. G. Jordan, M. Keene, C. M. Muirhead, A. I. M. Rae, N. Thomas, J. S. Abell, and S. Sutton, "Flux-Quantization in a High-Tc Superconductor," Nature 326 (6116), 855-855 (1987)]

It was also shown that the pairs involve a spin singlet state through measurements of the Knight shift in the superconducting state. The interaction between the electron spin $\vec{S}$ and the nuclear moment $\vec{I}$ is $\mathcal{H}_{int} = \vec{S} \cdot \vec{I}$, leading to the Knight shift $K(T)$ that measures the electron spin susceptibility. This is observed to go to zero in the limit of zero temperature, consistent with a spin singlet pairing state. [M. Takigawa, A. P. Reyes, P. C. Hammel, J. D. Thompson, R. H. Heffner, Z. Fisk, and K. C. Ott, "Cu and O NMR studies of the magnetic properties of $YBa_2Cu_3O_{6.63}$ ($T_c$=63 K)," Phys Rev B 43 (1), 247-257 (1991)]

# The Cuprate Fermi Surface is Cylindrical in Nature




Suchitra E. Sebastian, N. Harrison, P. A. Goddard, M. M. Altarawneh, C. H. Mielke, Ruixing Liang, D. A. Bonn, W. N. Hardy, O. K. Andersen, and G. G. Lonzarich, "Compensated electron and hole pockets in an underdoped high-$T_c$ superconductor," Phys Rev B **81 (21), 214524 (2010).**



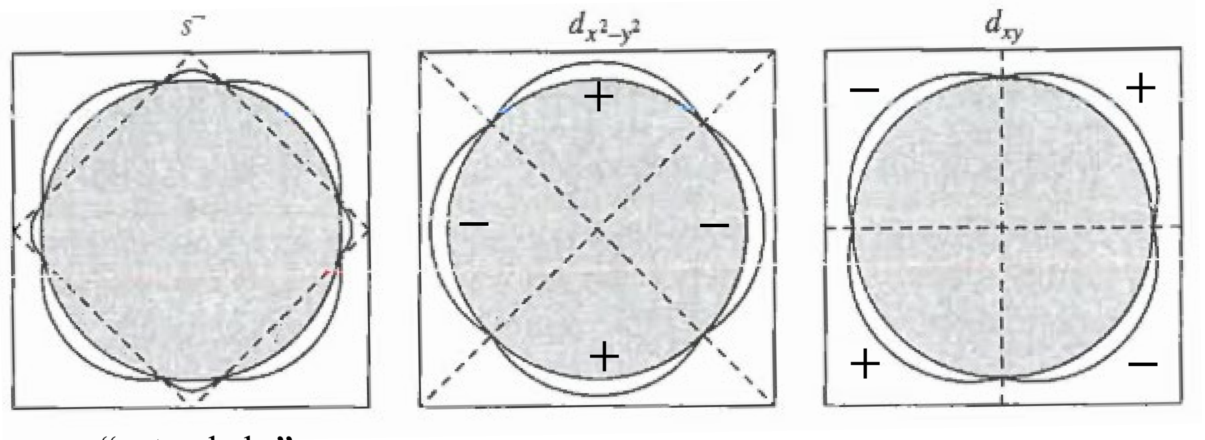

https://arpes.stanford.edu/research/quantum-materials/cuprate-superconductors

I. M. Vishik *et al.*, ***PNAS* 109, 18332 (2012)**

# Possible Order Parameters in Cuprate Superconductors

$$\Delta_{\mathbf{k}} = \Delta \qquad (s),$$
$$\Delta_{\mathbf{k}} = \Delta(\cos(k_x a) + \cos(k_y a))/2 \qquad (s^-),$$
$$\Delta_{\mathbf{k}} = \Delta(\cos(k_x a) - \cos(k_y a))/2 \qquad (d_{x^2-y^2}),$$
$$\Delta_{\mathbf{k}} = \Delta \sin(k_x a) \sin(k_y a), \qquad (d_{xy}),$$

$a$ =lattice constant in $CuO_2$ plane




"extended s"

$\Delta_{\hat{R}\vec{k}} = \Delta_{\vec{k}}$

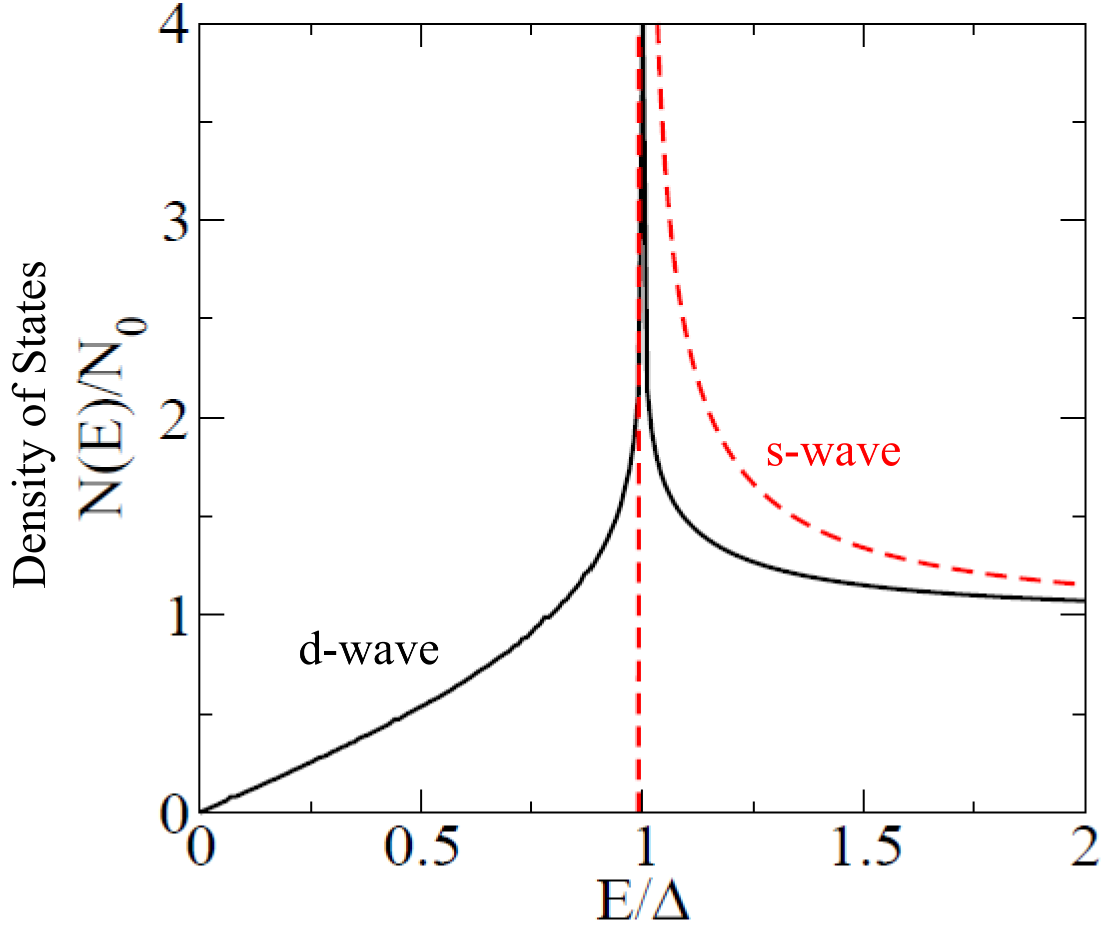
Unconventional superconductors

$\Delta_{\hat{R}\vec{k}} \neq \Delta_{\vec{k}}$

**Fig. 7.7** Three possible superconducting phases of a tetragonal superconductor, such as the high $T_c$ cuprates. The first, extended-$s$, has the full lattice symmetry. As shown it can have eight nodes of the gap on the Fermi surface. The remaining two functions $d$-$x^2-y^2$ and $d$-$xy$ both have four gap nodes on the Fermi surface. Much experimental evidence now points to the $d$-$x^2-y^2$ state as the one which is present in the high $T_c$ cuprate superconductors.

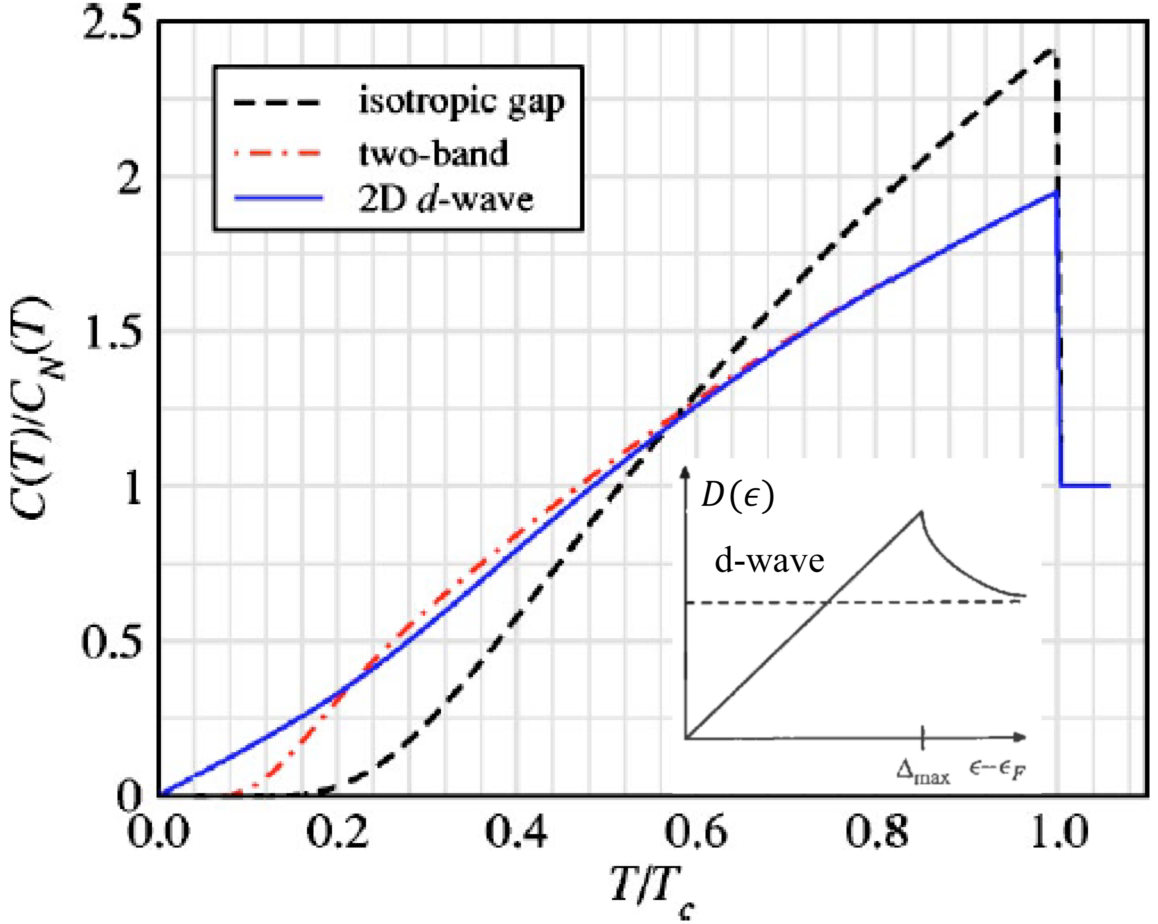
James F. Annett, *Superconductivity, superfluids, and condensates. (Oxford University Press, Oxford, 2004).*

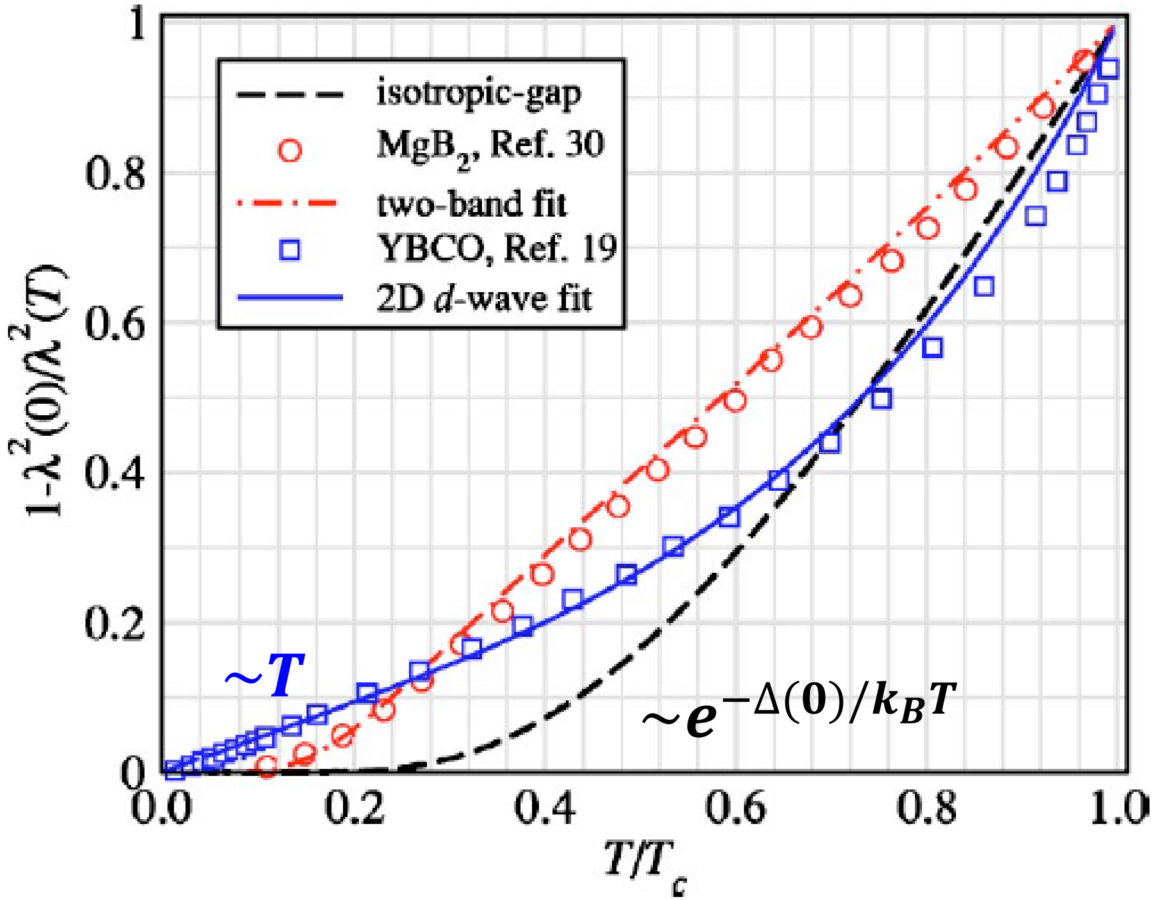
# Density of States in s-wave and d-wave Superconductors




Hyekyung Won, Stephan Haas, David Parker, Sachin Telang, András Ványolos, and Kazumi Maki, "BCS theory of nodal superconductors," AIP Conf Proc **789 (1), 3-43 (2005).**
https://doi.org/10.1063/1.2080347

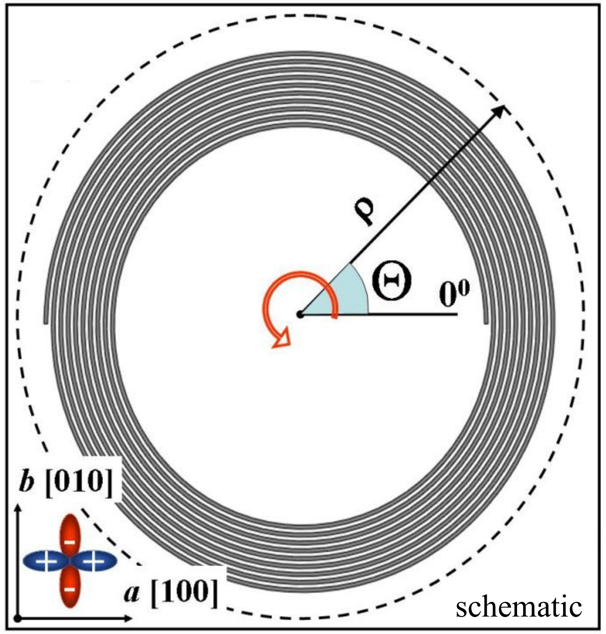
# Electronic Heat Capacity and Superfluid in s-wave and d-wave Superconductors

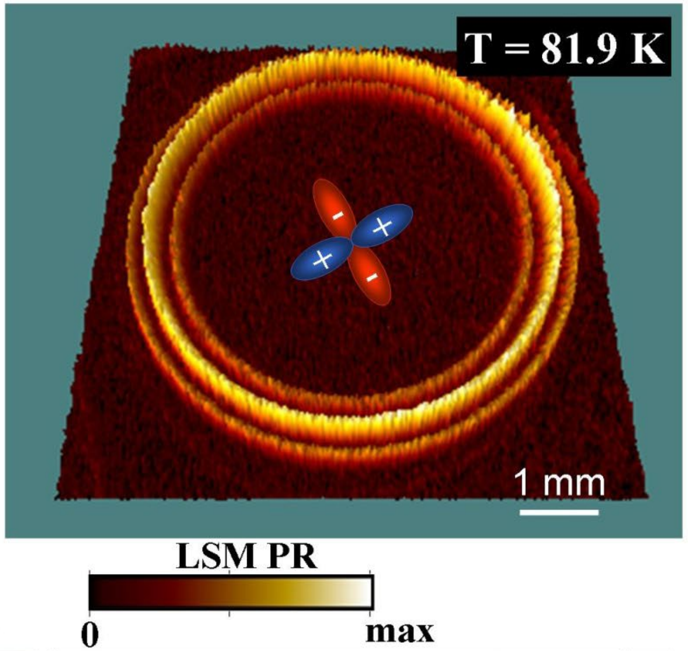


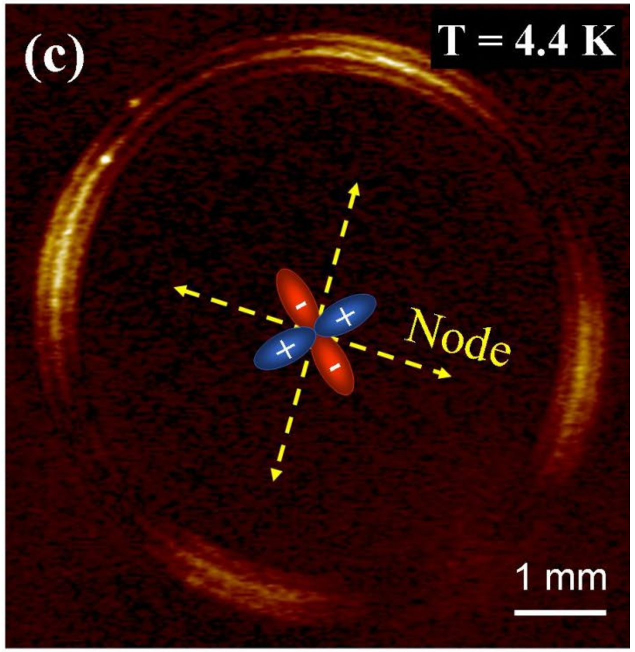

T. M. Mishonov, S. I. Klenov, and E. S. Penev, "Temperature dependence of specific heat and penetration depth of anisotropic-gap Bardeen-Cooper-Schrieffer superconductors for a factorizable pairing potential," Phys Rev B **71 (2), 024520 (2005).**

# LSM Imaging Results on YBCO Spiral Resonator



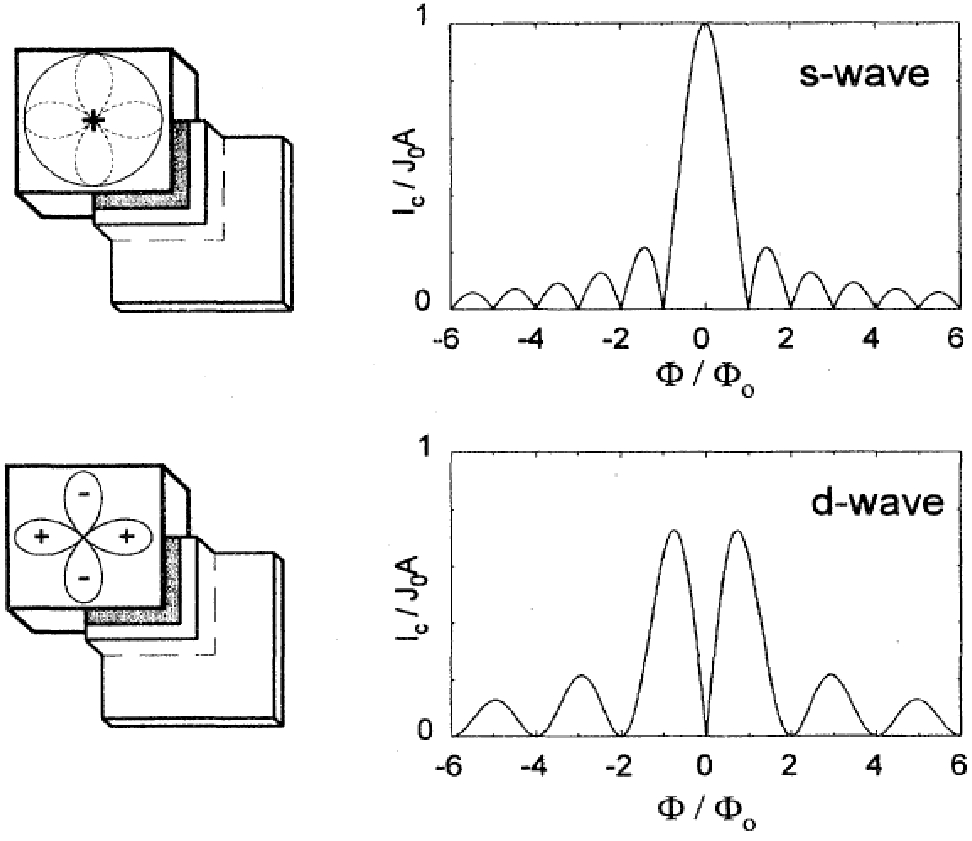



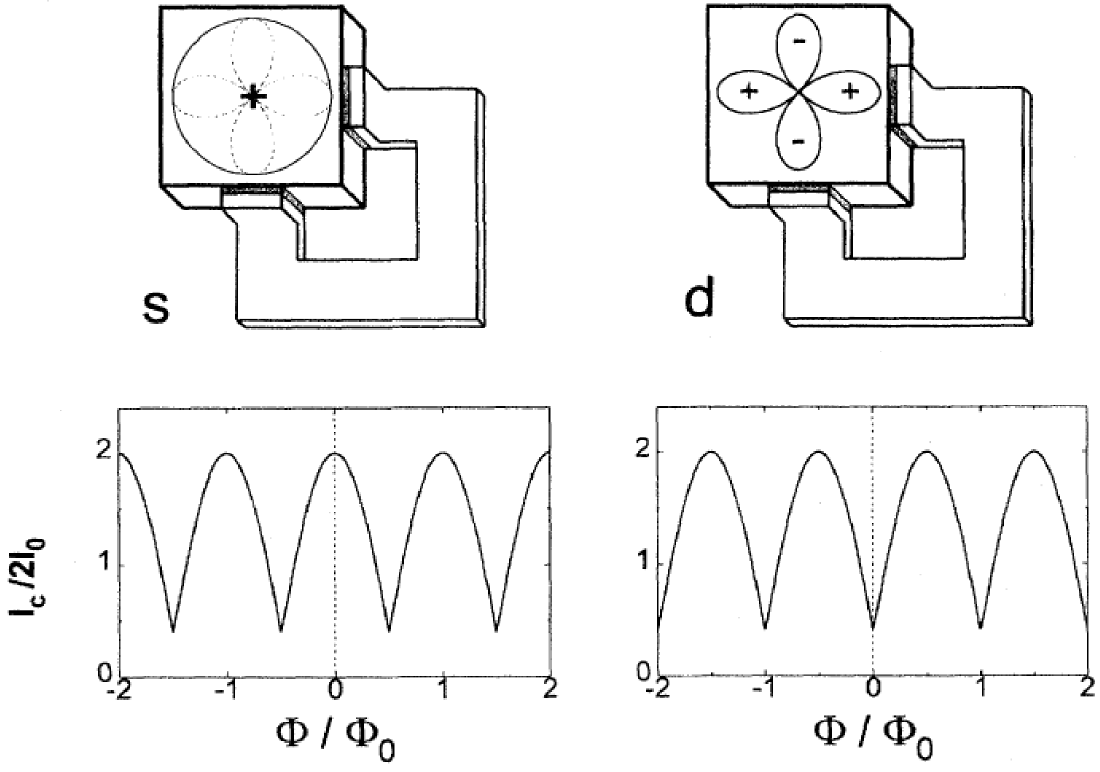

**Epitaxial $YBa_2Cu_3O_{7-\delta}$ on $LaAlO_3$**




Alexander P. Zhuravel, B. G. Ghamsari, C. Kurter, P. Jung, S. Remillard, J. Abrahams, A. V. Lukashenko, Alexey V. Ustinov, and Steven M. Anlage, "Imaging the Anisotropic Nonlinear Meissner Effect in Nodal $YBa_2Cu_3O_{7-\delta}$ Thin-Film Superconductors," Phys Rev Lett **110 (8), 087002 (2013)**

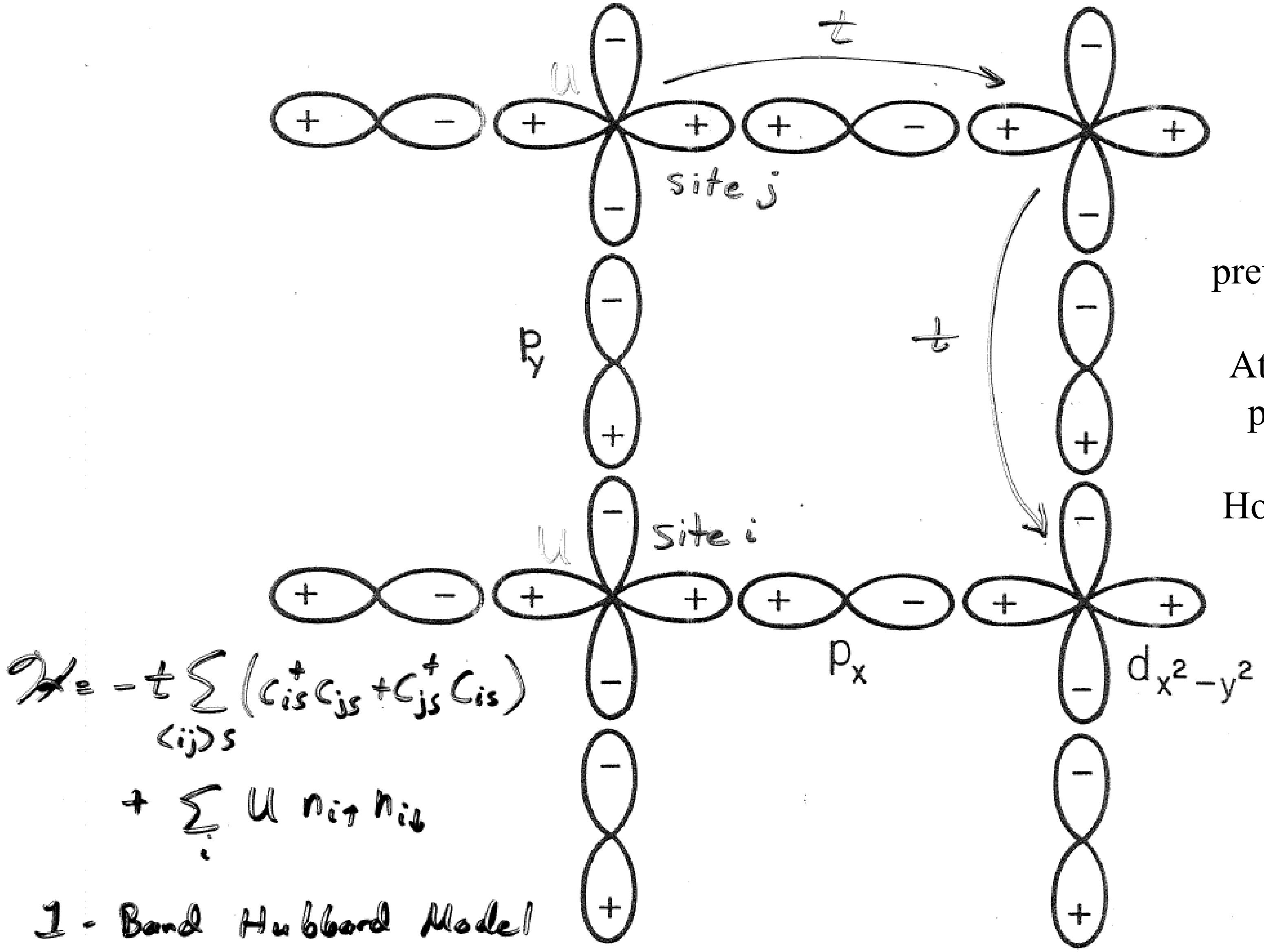
# Phase-sensitive tests of the symmetry of the pairing state in Cuprate Superconductors




D. J. Van Harlingen, "Phase-sensitive tests of the symmetry of the pairing state in the high-temperature superconductors—Evidence for $d_{x^2-y^2}$ symmetry," Rev Mod Phys **67 (2), 515-535 (1995)**

# 1-Band Hubbard Model of the $CuO_2$ Planes

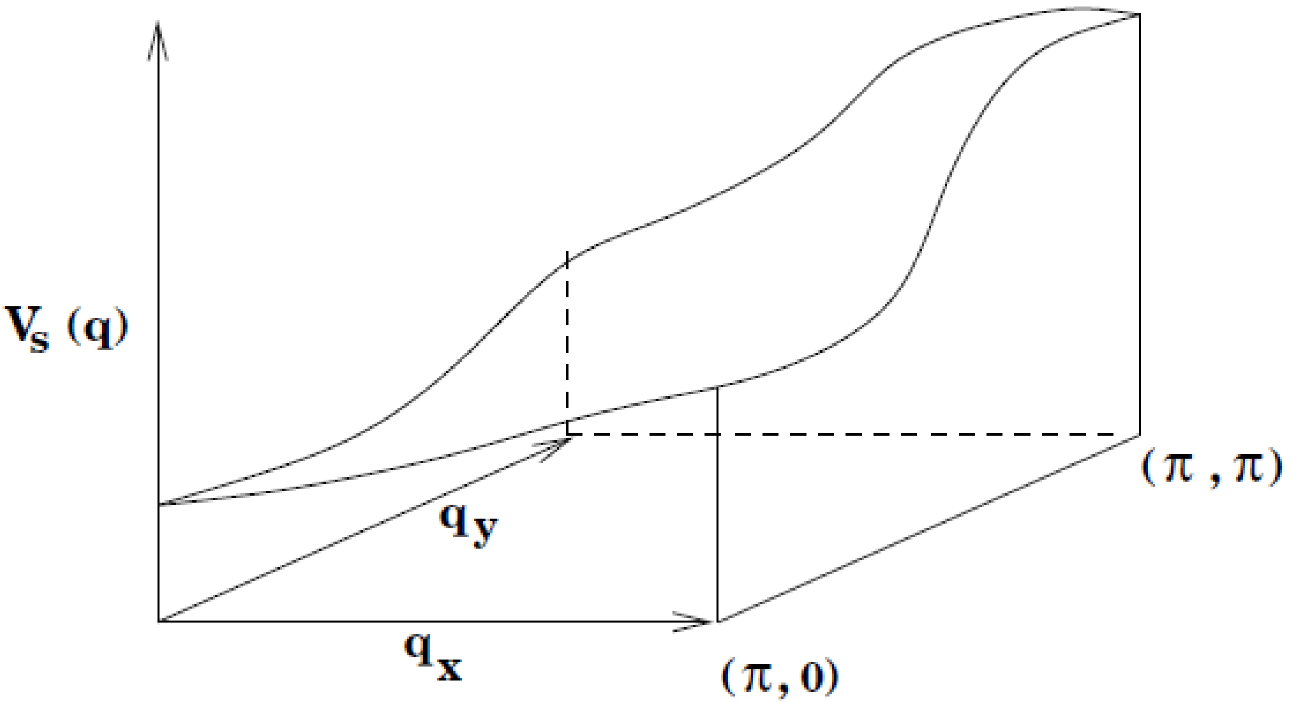



Typically $U >> t$ preventing double-occupation

At half-filling (one electron per site) there is AF LRO

How does the system evolve as holes are added?

D. J. Scalapino, "Superconductivity and Spin Fluctuations," J Low Temp Phys 117, 179-188 (1999);
D. J. Scalapino, "A common thread: The pairing interaction for unconventional superconductors," Rev Mod Phys 84 (4), 1383-1417 (2012)




$$V_s(q,\omega) \cong \frac{3}{2}\,\frac{\bar{U}^2\chi_0(q,\omega)}{1-\bar{U}\chi_0(q,\omega)}$$

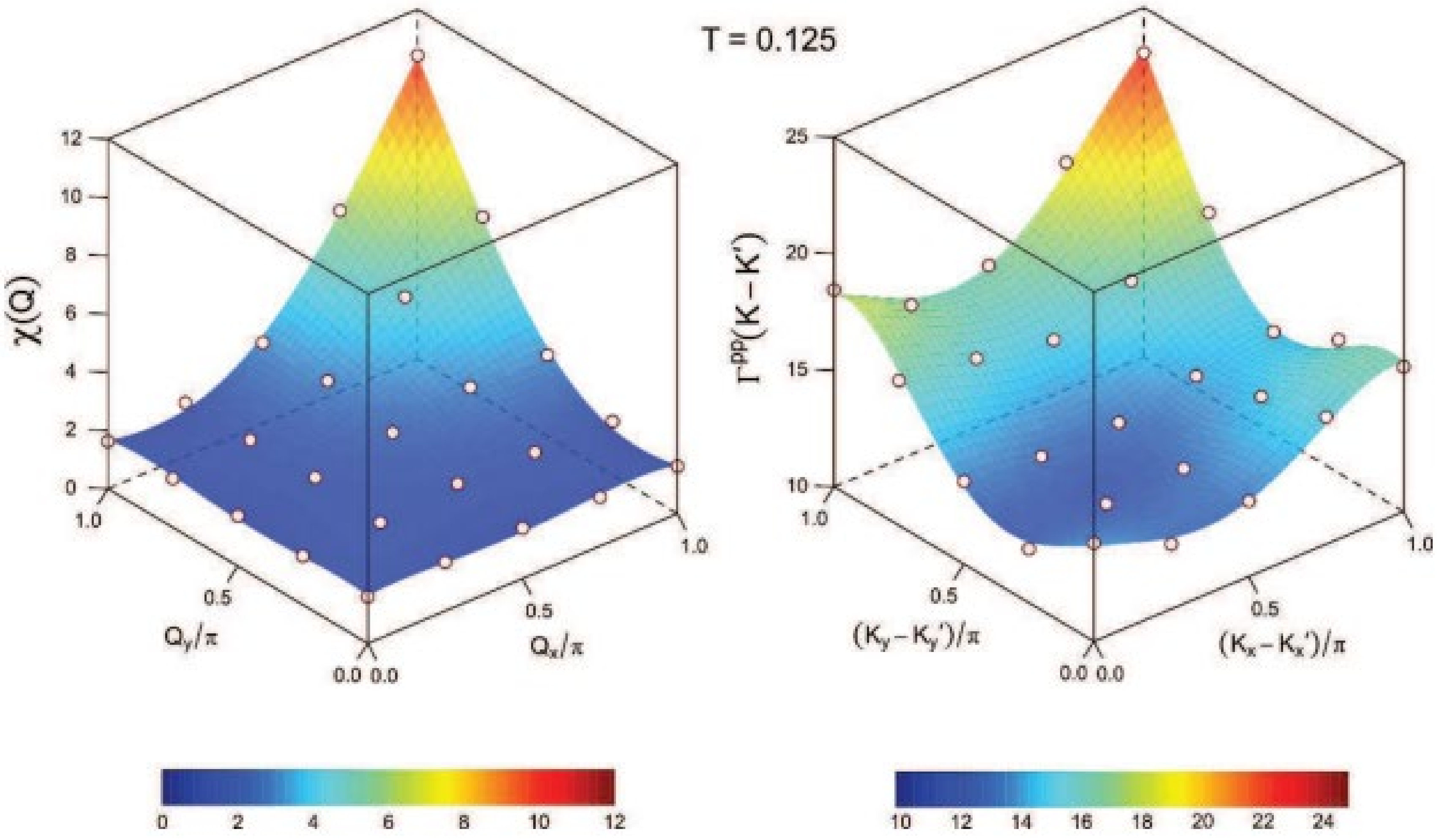
Purely repulsive

Fig. 4. Sketch of $V_s(q)$ versus $q$ for a two-dimensional system with short-range antiferromagnetic spin fluctuations.


https://doi.org/10.48550/arXiv.cond-mat/9908287

Douglas J. Scalapino, "Superconductivity and Spin Fluctuations," J Low Temp Phys **117, 179-188 (1999).**

# Cuprate Superconductor Spin Susceptibility and Pairing Interaction




D. J. Scalapino, "A common thread: The pairing interaction for unconventional superconductors," Rev Mod Phys **84 (4), 1383-1417 (2012).**

# Cuprate Pairing Interaction in Real Space

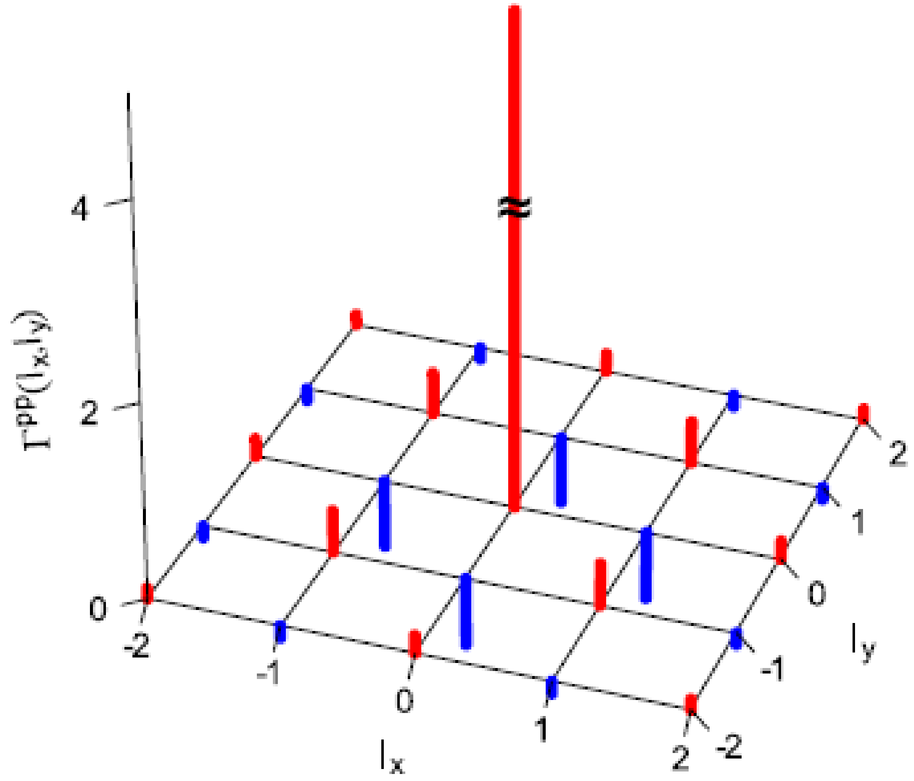



FIG. 19 (color online). The real space structure of the pairing interaction obtained from the Fourier transform Eq. (11) of $\Gamma^{pp}(k,k')$ at a temperature $T = 0.125t$ for $U = 4t$ and $\langle n \rangle = 0.85$. Here there is an attractive pairing interaction for a singlet formed between an electron at the origin and a near-neighbor site. The peak in $\Gamma^{pp}$ shown in Fig. 18 leads to a pairing interaction which oscillates in space.

$$V_s(\ell) = \sum_q e^{i\vec{q}\cdot\vec{\ell}} V_s(q, \omega = 0)$$

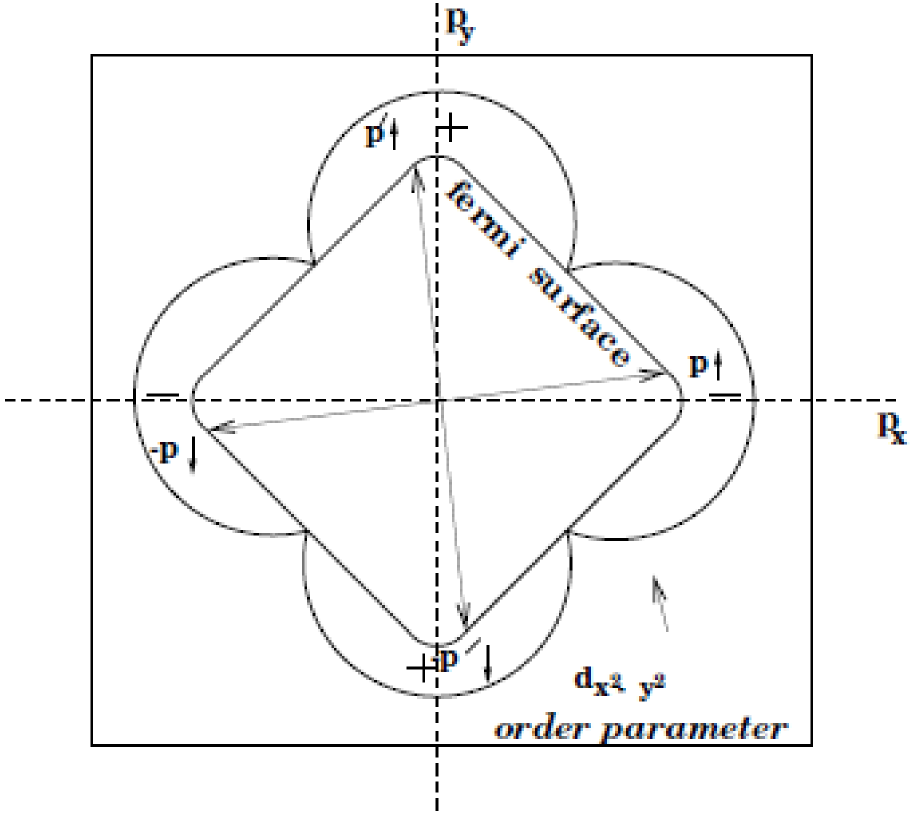
Fig. 6. Fourier transform of the singlet-pairing interaction $V_s(q)$ of Fig. 4 arising from the short-range antiferromagnetic spin fluctuations on a square lattice. Here one member of the singlet pair is located at the origin and the other at a surrounding site $\ell$. The potential is strongly repulsive for both electrons on the same site, as shown by the large positive bar at the origin. However, the potential is attractive on near-neighbor sites.

https://doi.org/10.48550/arXiv.cond-mat/9908287

Douglas J. Scalapino, "Superconductivity and Spin Fluctuations," J Low Temp Phys **117, 179-188 (1999).**

D. J. Scalapino, "A common thread: The pairing interaction for unconventional superconductors," Rev Mod Phys **84 (4), 1383-1417 (2012).**

# Superconductivity from the Repulsive Interaction




$$\Delta_{\vec{k}} = -\sum_{\vec{k}'} V_{\vec{k},\vec{k}'} \frac{\Delta_{\vec{k}'}}{2\sqrt{\xi_{\vec{k}'}^2 + \Delta_{\vec{k}'}^2}}$$

Recall that $V_{\vec{k},\vec{k}'}$ is peaked at the corners of the BZ.

Switch the sign of the gap $\Delta_{\vec{k}}$ for sides of the FS that differ by momentum transfer $\vec{q} = \pm\left(\frac{\pi}{a}, \frac{\pi}{a}\right)$.

The $d_{x^2-y^2}$ gap function is a self-consistent solution to the gap equation with this purely repulsive $V_{\vec{k},\vec{k}'}$

Fig. 5. Illustration showing how a $d$-wave gap can provide a solution of the BCS gap eq. (5) for a pairing interaction which increases at large momentum transfer like the type illustrated in Fig. 4.

https://doi.org/10.48550/arXiv.cond-mat/9908287

Douglas J. Scalapino, "Superconductivity and Spin Fluctuations," J Low Temp Phys **117, 179-188 (1999).**